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E0 Transitions in ^{146}Sm

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Abstract: The potential energy surfaces V(β, γ) for ^{146}Sm, levels energy belonging to the ground state (g.s), beta (β) , gamma (γ) bands, electromagnetic transition probabilities B(E2) 's , mixing ratios $\delta(E2/M1)$ and strength of the electric monopole, (E0), transitions X(E0/E2) are calculated within the frame work of the interacting boson approximation model (IBA-2). The results of these calculations are compared to the available experimental data and reasonable agreement has obtained.

Keywords: Interacting boson approximation model, Levels energy, Transition probabilities, Mixing ratios, Electric monopole transitions

1. Introduction

The even-even samarium isotopes are of considerable interest because of the phase transition from spherical to deformed shape. This transition has attracted many authors to study that series of isotopes theoretically and experimentally.

The theoretical works that discussed spectral statistics of the rare-earth nuclei either using quantum mechanical effects to see the effects of the half-life of α-decay of spherical isotopes [1] or shell model configuration effects and compare to the available experimental data[2]. Also, interacting boson approximation Hamiltonian [3-6] and finite amplitude method [7] are used in calculating the energy density and the electric monopole strength in the chain of samarium isotopes.

Experimentally the lifetime of levels in ^{146}Sm has measured [8] using (^{11}B, 4n) reaction at 54 MeV on natural La target evaporated on gold. The results are compared to the calculations of the IBA model and cluster – vibrator model. The calculated values of the two models are not inconsistent with the observed transition probabilities up to $6^+ \longrightarrow 4^+$ transition and failed to reproduce that for $8^+ \longrightarrow 6^+$ transition.

Unfortunately scarce data are available about ^{146}Sm and the interacting boson approximation model IBA-2 has been used in calculating levels energy, transition probabilities, potential energy surfaces V(β, γ), mixing ratios $\delta(E2/M1)$ and strength of the electric monopole, (E0), transitions X(E0/E2).

2. Interacting boson approximation model (IBA-2)

2.1 Levels energy:

IBA-2 model [9-11] has been applied to the positive-parity of the low - lying states in ^{146}Sm. The Hamiltonian employed for the present calculation is given [12] as:

$$H = \varepsilon_\pi (d^+ \times d)_\pi + \varepsilon_\nu (d^+ \times d)_\nu + V_{\pi\pi} + V_{\nu\nu} + KQ_\pi . Q_\nu + M_{\pi\nu} \quad (1)$$

Where ε_π, ε_υ are the, (proton), π and ,(neutron), υ energies respectively and they are assumed equal

$$\varepsilon_\upsilon = \varepsilon_\pi = \varepsilon$$

The third and fourth term of equation. (1) represent the π - υ interactions operator which is given by :

$$V_{\rho\rho} = \sum_{L=0,2,4} \frac{1}{2} C_{L_\rho} (2L+1)^{\frac{1}{2}} \left[(d^+ x d^+)^L_\rho . (d\ x\ d)^L_\rho \right]^{(0)}$$

$$\rho = \pi \text{ or } \upsilon \quad (2)$$

The fifth term is the quadrupole operator which is given by the usual expression, (K is the strength of the π and υ bosons quadrupole interactions),:

$$Q_\rho = \left[(s^+ x\ d) + (d^+ x\ s) \right]_\rho + x_\rho (d^+ x\ d)_\rho$$

$$\rho = \pi \text{ or } \upsilon \quad (3)$$

The sixth term is the Majorana operator and is given by :

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$$M_{\Pi\nu} = -2\sum_{k=1,3}\zeta_k\,(d^\dagger_\Pi x d^\dagger_\nu).(d_\Pi x d_\nu) + \zeta_2(d^\dagger_\Pi x s^\Pi_\nu - s^\dagger_\Pi x d^\dagger_\nu)^{(2)}.(d_\Pi x s_\nu - s_\Pi x d_\nu) \quad (4)$$

2.2 Reduced transition probabilities, B (E2)'s:

The electric quadrupole transition operator [9] employed in this study is defined as :

$$T^{(E2)} = e_\pi Q_\pi + e_\upsilon Q_\upsilon \quad (5)$$

Where:

$T^{(E2)}$:absolute transition probability of the electric quadrupole (E2) transition,

e_π and e_υ : the π and υ effective charges, and

Q_ρ : the quadrupole operators which is the same as that in equation.(3)

$\rho = \pi$ or υ .

The reduced electric quadrupole transition rates between $I_i \rightarrow I_f$ states is given by

(6)

$$B(E2, I_i - I_f) = \left[\langle I_f \| T^{(E2)} \| I_i \rangle \right]^2 \big/ (2I_i + 1)$$

where

I_i : the initial state of the electric quadrupole transition, and

I_f : the final state of the electric quadrupole transition.

The proton and neutron boson numbers N_π and N_υ respectively can be treated as parameters. They are fixed to be half the number of valence fermions and counted from the nearest closed shell.

The effective charges e_π and e_υ depend on the total number of bosons N, number of υ bosons N_υ , number of π bosons N_π and the experimental value of $B(E2, 2_1^+ \rightarrow 0_1^+)$, where

$$N = N_\pi + N_\nu \quad (7)$$

The effective charges of the vibrational limit are calculated [13] using:

$$(N_\pi)^{-1}[(N)(N+3)^{-1} B(E2, 2_1^+ \rightarrow 0_1^+)]^{1/2} = e_\pi + e_\upsilon N_\upsilon / N_\pi \quad (8)$$

3. Results and discussion

3.1 The potential energy surfaces:

The potential energy surfaces, V(β, γ), for ^{146}Sm nucleus as a function of the deformation parameters β and γ has been calculated using [14] equation:

$$E N_\upsilon N_\pi(\beta, \gamma) = \langle N_\pi N_\upsilon; \beta\gamma \mid H_{np} \mid N_\pi N_\upsilon; \beta\gamma \rangle$$

$$= \varepsilon_d (N_\upsilon + N_\pi)\beta^2(1+\beta^2)^{-1} + \beta^2(1+\beta^2)^{-2}\{ k N_\pi N_\upsilon[4-(\bar{X}_\Pi + \bar{X}_\nu)\beta\cos 3\gamma + \bar{X}_\Pi \bar{X}_\nu \beta^2] + N_\upsilon(N_\upsilon-1)\left(\frac{C_o}{10} + \frac{C_2}{7}\right)\beta^2\} \quad (9)$$

where

$$\bar{X}_\rho = (2/7)^{0.5} X_\rho \qquad \rho = \pi \text{ or } \upsilon \quad (10)$$

The contour plot of the potential energy surfaces is pres in **Figure 1**, while the plot for the potential energy surfaces versus the deformation parameters β and γ for γ=0º (prolate) and γ=60º (oblate) is presented in Figure 2. It is clear from the graphs that ^{146}Sm nucleus has vibrational character since vibrational character require:

(a) Minimum surface energy at deformation β=0, and
(b) Circular contours around β=0.

The two requirements are exist for the nucleus as shown in figures 1,2. The previous arguments support the assumption made by other authors that ^{146}Sm is a vibrational-like nucleus.

The value of the deformation parameter β which is corresponding to the minimum value of the potential energy implies a decrease of the moment of inertia, and in turn to an increase of the spacing between levels in the ground state and β bands.

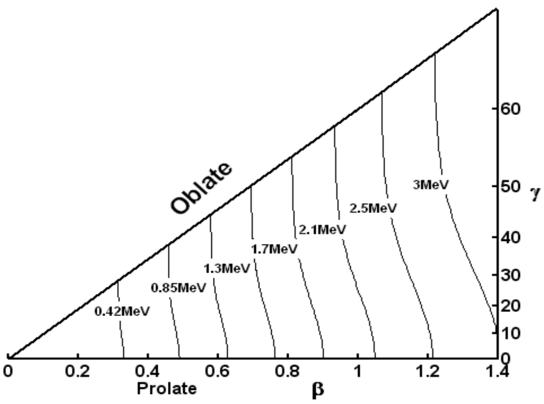

Figure 1:. Contour plot of the potential ergy surfaces

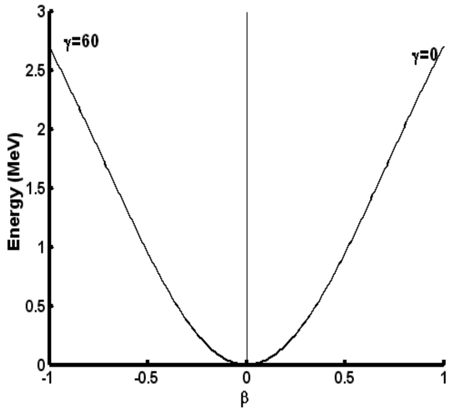

Figure 2.: The potential deformation energy (MeV) versus the deformation β.(γ =0◦ prolate and γ =60◦ oblate)

3.2 Levels energy:

The computer codes NPBOS and NPBEM [15] of the IBA-2 have been used in calculating energy eigenvalues, wave functions and E2 transition matrix elements. The Hamiltonian parameters of equation (1) used in the present calculation are:

ε = 0.890 K = -0.080 $X_{¶}$ = -1.300 X_{υ} = -1.400
$\varepsilon_1 = \varepsilon_3$ = 0.090 ε_2 = 0.100
$C_{L\nu}$ = 0.000, 0.050, 0.000
$C_{L\pi}$ = 0.000, 0.100, 0.000 (all in MeV).

The results of the calculation of energy levels together with the experimental data are displayed in **Figure 3**.

The calculation is limited to states of spin- parity up to $I=8^+$ and the energies of seventeen levels were reproduced and in general, reasonable agreement has achieved between the experimentally measured [16] and the calculated ones .

3.3 Reduced transition probabilities, B (E2)'s :

The π and υ effective charges $e_{¶}$ and e_{υ} were calculated using the vibrational limit [13]. The calculated values are :

$e_{¶}$ = 0.231 e.b and e_{υ} = 0.531 e.b

The calculated and the experimental B(E2) values are listed and compared in **Table 1**. There is only one experimental value available [17] which is $B(E2,0_1^+-2_1^+)= 0.48(8)\ e^2b^2$. It is based on the life time measurement of 747.1 keV level which is < 7.2 ps.

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The quasi ground state , β and γ – bands are presented in spectra columns as in Figure 4. The calculated transition probability of the internal gamma rays in any band have greater values than the gamma transitions in inter bands have, **Table 1**.

These energy and ratios of different energies to those of the first excited 2_1^+ state are listed in Table 2 and show vibrational character to ^{146}Sm nucleus.

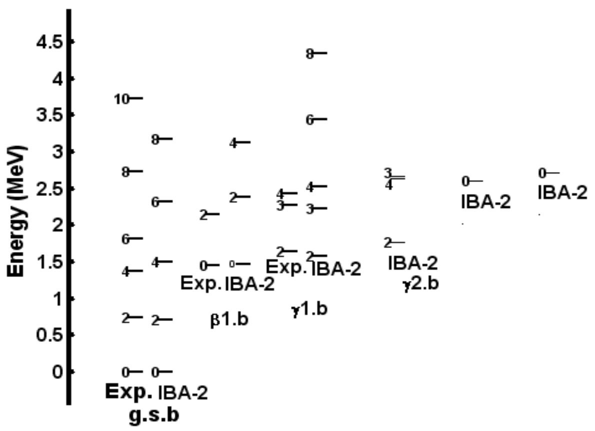

Figure 4: Comparison between the experimental and theoretical energy levels

Table 1 : Reduced transition probability values , B (E2) (in e^2b^2)

I_i^+	I_f^+	B(E2) Exp*	B(E2) IBA-2	I_i^+	I_f^+	B(E2) Exp*	B(E2) IBA-2
2_1	0_1	0.090	0.1065	3_2	4_1		0.0082
2_1	0_2		0.0321	3_2	4_2		0.0515
2_1	0_3		0.0012	3_2	4_3		0.0019
2_1	0_4		0.0001	3_2	4_4		0.0088
2_2	0_1		0.0015	4_1	2_1		0.1718
2_2	0_2		0.0109	4_1	2_2		0.0008
2_2	0_3		0.0067	4_1	2_3		0.0233
2_2	0_4		0.0095	4_1	2_4		0.0247
2_3	0_1		0.0083	4_2	2_1		0.0003
2_3	0_2		0.0383	4_2	2_2		0.0649
2_3	0_3		0.0000	4_2	2_3		0.0270
2_3	0_4		0.0118	4_2	2_4		0.0130
2_4	0_1		0.0003	4_3	2_1		0.0099
2_4	0_2		0.0650	4_3	2_2		0.0005
2_4	0_3		0.0058	4_3	2_3		0.0327
2_4	0_4		0.0132	4_3	2_4		0.0160
2_1	2_2		0.1058	4_4	2_1		0.0001
2_1	2_3		0.0111	4_4	2_2		0.0021
2_1	2_4		0.0000	4_4	2_3		0.0006
2_2	2_1		0.1058	4_4	2_4		0.0187
2_2	2_3		0.0243	4_1	3_1		0.0311
2_2	2_4		0.0319	4_1	3_2		0.0063
2_3	2_1		0.0111	4_2	3_1		0.0140
2_3	2_2		0.0243	4_2	3_2		0.0401
2_3	2_4		0.0134	4_3	3_1		0.00 83
2_4	2_1		0.0000	4_3	3_2		0.0015
2_4	2_2		0.0319	4_4	3_1		0.0607
2_4	2_3		0.0134	4_4	3_2		0.0068
3_1	2_1		0.0039	4_2	4_1		0.0589
3_1	2_2		0.1462	4_3	4_1		0.0000
3_1	2_3		0.0074	4_3	4_2		0.0034
3_1	2_4		0.0056	4_4	4_3		0.0250
3_2	2_1		0.0034	4_4	4_2		0.0483
3_2	2_2		0.0161	4_4	4_1		0.0006
3_2	2_3		0.0545	6_1	4_1		0.2080
3_2	2_4		0.0585	6_1	4_3		0.0247
3_1	4_2		0.0180	6_1	4_4		0.0095
3_1	4_3		0.0107	6_2	4_1		0.0006

[17]

Table 2: The levels energy ratios

Quantity	Exp*.	IBA-2	SU(5)	O(6)	SU(3)
$E2_2^+/E2_1^+$	2.20	2.21	2.00	2.50	3.33
$E4_1^+/E2_1^+$	1.84	2.09	2.00	2.50	3.33
$E6_1^+/E2_1^+$	2.42	3.24	3.00	4.50	7.00
$E4_2^+/E2_1^+$	3.05	3.53	3.00	4.50	7.00
$E3_1^+/E2_1^+$	3.03	3.11	3.00	4.50	7.00
$E8_1^+/E2_1^+$	3.66	4.45	4.00	7.00	12.00
$E4_3^+/E2_1^+$	-	3.70	4.00	7.00	12.00

[18]

SU (5) = vibrational limit,
O (6) = gamma unstable limit, and
SU (3) = rotational limit.

3.4 Multipole mixing ratio $\delta(E2/M1)$:

The reduced electric E2 and magnetic M1 matrix elements of different gamma ray transitions were calculated and used in calculating multipole mixing ratios using [19]

$$\delta(E2/M1) = 0.835 E_{\gamma}(MeV)\frac{\prec|I_f\,\|T^{E2}\|\,I_i|\succ}{\prec|I_f\,\|T^{M1}\|\,I_i|\succ} \quad (11)$$

The calculated values are presented in Table 3 in comparison to the available experimental ones and reasonable agreement has achieved.

Table 3: Mixing ratios (δ) for ^{146}Sm

I_i^+	I_f^+	Exp.(*)	IBA-2
2_2	3_1	0.33±0.04	0.47
2_3	3_1		-2.42
3_1	2_2		-1.47
3_2	2_3		2.82
3_2	3_1		1.87
3_2	4_1	-0.10^{+20}_{-26}	0.95
4_4	4_2	0.79^{+29}_{-24}	1.27

*[16]

3.5 Strength of the electric monopole transitions :

The electric monopole transitions, E0, are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. It is a pure penetration effect where it caused by the electromagnetic interaction between the nuclear charge and the atomic electron which penetrates the nucleus. The E0 strength can be considered as the ratio between the reduced transition probability of competing E0 and electric quadrupole, E2, transitions de-populating the same level.
The strength of the electric monopole transition, (E0 / E2), can be determined by [20]:

$$X_{if'f}(E0/E2) = \frac{B(E0, I_i - I_f)}{B(E2, I_i - I_{'f})} \quad (12)$$

$$X_{if'f}(E0/E2) = 2.54x10^{9}xA^{4/3}\frac{(E_{\gamma}MeV)^5}{\Omega_{K,L}}$$

$$\alpha(E2)\frac{T_e(E0, I_i - I_f)}{T_e(E2, I_i - I_{'f})} \quad (13)$$

where
B(E0, I_i - I_f) : reduced transition probability of the E0 transition,
B (E2, I_i – I'_f) : reduced transition probability of the E2 transition.
A : mass number ,
I_i : spin of the initial state where E0 & E2 transition are depopulating it,
I_f : spin of the final state of E0 transition,
I'_f : spin of the final state of E2 transition,
E_{γ} ;gamma ray energy,
Ω_{kl} : electronic factor for K,L shells [21],
α (E2) : conversion coefficient of the E2 transition,
T_e (E0, I_i -I_f): absolute transition probability of the E0 transition between I_i and I_f states, and
T_e (E2,I_i–I'_f) : absolute transition probability of the E2 transition between I_i and I'_f states.

The calculated strength of ten electric monopole transitions for $\Delta I^+=0$, $I_i^+=0$, $I_f^+=0$ and $I_{'f}^+$ =2 are presented in Table 4. The large value for (0_3^+ – 0_2^+ - 2_3^+) may be due to the vanishing B(E2) value rather than the large B(E0) value. Unfortunately there is no experimental data for comparison.

Table 4. $X_{iff'}(E0/E2)$ ratios for E0 transitions

I_i^+	I_f^+	$I_{f'}^+$	Exp	IBA-2	I_i^+	I_f^+	$I_{f'}^+$	Exp	IBA-2
0_2	0_1	2_1		0.0057	0_4	0_3	2_2		0.0010
0_3	0_2	2_3		2.9032	0_4	0_3	2_1		0.1081
0_3	0_2	2_2		0.0431	0_4	0_2	2_4		0.0034
0_3	0_2	2_1		0.2439	0_4	0_2	2_3		0.0038
0_3	0_1	2_3		1.3548	0_4	0_2	2_2		0.0047
0_3	0_1	2_2		0.0201	0_4	0_2	2_1		0.3783
0_3	0_1	2_1		0.1138	0_4	0_1	2_4		0.0012
0_4	0_3	2_4		0.0009	0_4	0_1	2_3		0.0013
0_4	0_3	2_3		0.0011	0_4	0_1	2_2		0.0016
0_4	0_3	2_2		0.0013	0_4	0_1	2_1		0.1351

4. Conclusions

1. The contour plot of the potential energy surfaces V(β, γ) shows that ^{146}Sm is a spherical nucleus and has a vibrational character.
2. The vibrational limit of the IBA-2 has been used and energies of seventeen-levels were calculated.
3. The reduced transition probability of gamma transitions were calculated and normalized to B(E2,$0_1^+ \rightarrow 2_1^+$) = 0.48(8).
4. The mixing ratios and the strength of twenty electric monopole (E0) transitions was calculated for the first time.

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