

Heuristic Algorithms to Solve Cost-Oriented Line Balancing Problem

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Abstract: In many real world assembly line systems which the work-piece is of large size more than one worker work on the same work-piece in each station. This type of assembly line is called multi-manned assembly line (MAL). In the classical multi-manned assembly line balancing problem (MALBP) the objective is to minimize the manpower needed to manufacture one product unit. Apart from the manpower, other cost drivers like wage rates or machinery are neglected in this classical view of the problem. However due to the high competition in the current production location, sinking the production costs and increasing utilization of available resources are very important issues for manufacturing managers. In this paper a cost-oriented approach is used to model the MALBP with the aim of minimizing total cost per production piece. A mathematical model is settled to solve the problem. Since the proposed model is NP-hard, several heuristic algorithms are presented to efficiently solve the problem. Several examples are solved to illustrate the proposed model and the algorithms.

Keywords: Line balancing; Multi-manned assembly line; Cost-oriented approach; Heuristic.

1. Introduction

Assembly lines are flow-based production systems used to manufacture standardized production units in high volume. These systems even gain importance in manufacturing customized products in low volume. Assembly lines consist of workstations (also called stations) that are arranged along a conveyor belt or similar material handling system. In each station a set of tasks are performed on the work-piece. Beginning from the first station, each work-piece is moved from station to station with a constant transportation speed throughout the line. The production speed is determined by the cycle time which is the time between completions of two consecutive production units. The work content of each station in the line is constrained to be less than or equal to the cycle time. The total work needed to assemble the final product is divided into n basic operations $I = \{1, 2, \dots, n\}$, these elementary operations are called tasks. Each task j needs d_j^t units of time to be accomplished; this duration is called task time. Furthermore there are some precedence relations among tasks. Typically these relations are accessible in a precedence graph in which each vertex presents a task and each arc (i, j) presents a precedence relation between tasks i and j .

The problem of partitioning tasks to stations in order to optimize some objective functions is called assembly line balancing (ALB) problem. The most studied problem in the field of ALB is the simple assembly line balancing problem (SALBP) and has the following assumptions [1]-[3]: Mass production of one identical goods.

- Given production process.
- Paced line with a fixed cycle time C .

- Each task j has a Deterministic and numeral operation time d_j^t .
- No assignment limits alongside precedence constraints.
- Serial line layout with m stations.
- All stations are similarly equipped with reverence to machines and workers.
- Maximize the line efficiency: $Eff = \frac{t_{sum}}{m \times C}$ in which m is the number of stations and $t_{sum} = \sum_{j=1}^n d_j^t$ is the sum of processing time of all tasks.

These assumptions are very restricting with respect to real world assembly line systems. Therefore many researchers have focused on changing or releasing some of these assumptions to obtain more realistic models. The resultant problems are called generalized assembly line problem (GALBPs) [4].

Numerous generalizations have been considered for the ALBP. Some examples of these generalizations are considering U-shaped assembly lines balancing [5], parallel workstations [6], considering process alternatives [7] and two sided assembly lines [8]. Some latest surveys of generalized assembly line problems are [2], [3], [9], [10].

In many real-world assembly lines the production unit is of large size and there are more than one operator working on the same work-piece in each station. This situation is first identified and modeled by Dimitriadis [11]. In a MAL more than one operator can be working on the same work-piece in each station. This results in several advantages over simple assembly lines. Some samples of these gains are reducing the

length of the line and consequently reducing the work in process, reducing the costs of tools, machinery and transportation system [12].

Even though MALs are very common in real world assembly line production systems, only a small number of research papers have considered MALBPs. Dimitriadis, introduced the MALBP and presented a heuristic assembly line balancing procedure to solve the problem [11]. Cevikcan et al, developed a mathematical programming model to create assembly physical multi-manned stations in mixed model assembly lines. They also presented a scheduling-based heuristic to solve the problem [13]. Chang & Chang, proposed a mixed-model assembly line balancing problem with multi-manned workstations and developed a mathematical model for the mixed-model assembly line balancing problem with simultaneous production (MALBPS) to obtain the optimal number of workstations. They also presented a coding system, Four-Position Code (FPC), to re-code the tasks to tackle this issue, and provided a computerized coding program written in C++ to generate those FPCs [14]. Fattahi et al, developed a mathematical programming model for MALBP. They also proposed an ant colony meta-heuristic approach to solve the problem [12].

In the literature of MAL usually the objective is to minimize the number of workers for a given cycle time [12] or minimizing the idle time [13], [14]. However due to the high competition in the current production location, sinking the production costs and swelling utilization of available resources are very important issues for manufacturing supervisors. Therefore expanding a model to straight minimize the production costs is of significant interest. In this paper the MAL configuration is considered with a cost-oriented approach. Generally final assembly is a labour-intensive production [15]. In the cost oriented approach the objective is to minimize the total cost per product unit [16]-[19]. Therefore the significant cost drivers should be analyzed.

At first the labor costs are considered. The payment of a worker is dependent on the “job values” determined by the well-known work measurement systems [19]. In an assembly line there are tasks with different levels of difficulty and job values assigned to a worker. For each task i it is possible to consider a wage rate k_i^{pw} , $i \in I$ [MU/TU] (TU time unit, PU production unit, MU monetary unit) which is related directly to its job value. Wage rate of an operator working in a station along with other operators on the same work-piece is determined by the most difficult task assigned to him (or her) i.e. the task with the highest job value. Therefore wage rate of worker l in station j is:

$$k_{jl}^{pw} = \max\{k_i^{pw} | i \in I_{jl}^s\}, j = 1, 2 \dots J, l = 1, 2 \dots MC \text{ [MU/TU]}$$

where I_{jl}^s is the set of tasks assigned to worker l in station j and MC is the maximum feasible concentration of workers in each station. It is important to note that the wage rates are paid for the total cycle time and not only for the sum of duration of tasks performed by the worker.

Furthermore costs of capital should be considered. Examples of this kind of costs are machinery and material handling system e.g. conveyor. It is assumed that the costs of capital are directly dependent on the length of the line i.e. number of stations. The machinery needed to perform the operations can be special machinery to perform a special task or

universal machinery. The number of special machinery can be assumed to be fixed and independent of assignment of tasks to workers in stations. In addition it is assumed all of the stations need identical universal machines. Therefore the costs of capital for all stations are the same.

Other costs such as costs of material are assumed to be independent of the length of the line or assignment of tasks to stations [16]. Therefore the total costs per product unit k [MU/PU] can be formulated as $k = \sum_{i \in L} \sum_{j \in J} C \times k_{ji}^{pw} + m \times k^{sc}$ where k^{sc} [MU/PU] is the total cost of capital.

Reviewing the literature of cost-oriented assembly line balancing, Rosenberg and Ziegler, assumed that the operation of a station k causes a wage rate w_k per time unit equal to the maximum wage rate of all tasks that are assigned to that station. The objective is to minimize the aggregate wage rate over all stations, while the number of stations is variable. They described and evaluated priority rule based heuristics, where some of the rules are available for SALBP-1 [17]. Amen, extended the problem by considering the costs of capital e.g. cost of machinery or transportation system [19]. Amen [18] and Amen [19], proposed a branch and bound algorithm to solve the problem which applies a station-oriented construction method and laser search strategy. Amen developed station-oriented priority rule based procedures with cost-oriented dynamic priority rules and compares them to existing ones using a large set of problem instances which is generated randomly. The new rule named “best change of idle cost” had a better performance than all other rules [20], [21]. For the same problem, Amen concentrated on general model formulations that can be solved by standard optimization tools and introduced several improvements to existent models [15]. These models are planned for both general branch-and-bound techniques with LP-relaxation or general implicit enumeration techniques. They also discussed the solution difficulty of the problem and showed that the “maximally-loaded-station-rule” has to be replaced by the “two-stations-rule”; which causes an enormous increase in solution difficulty compared to the time-oriented version. Malakooti [22], [23] and Malakooti & Kumar [24] considered a multi-objective ALBP with objectives that are based on cost and capacity.

In this paper a cost-oriented approach is used to model the MAL which to the best of our knowledge hasn't been considered in the literature so far. Then different heuristics are proposed to solve the problem instances of large and medium size. The rest of this paper is organized as follows: in section 1 the proposed model is described and a mathematical formulation is developed to solve the problem. Seven heuristic algorithms are proposed to solve the problem in sections 2. Computational results are presented in section 3. Finally the chief conclusions of the paper and suggestions for future research are presented in section 4.

2. Proposed model and mathematical formulation

In this paper the paced assembly line with multi-manned workstations is considered which is very common in real world assembly lines but a small number of research papers have considered this type of assembly line. The work-piece

stays at each station for a certain amount of time called cycle time. In each station there are several workers performing different tasks on the same work-piece. Every employee starts the tasks given to him (or her) as soon as it is technically possible. The main objective in this type of assembly line is to reduce the length of line while maintaining the effectiveness of the line. This type of multiple workers working on the same work-piece at the same time requires the work-piece to be of large size e.g. vehicle final assembly. Traditionally in simple assembly lines all of the tasks assigned to a worker can be performed

continuously if the precedence relations are observed. But in multi-manned lines some tasks assigned to a worker may be delayed by the tasks assigned to other workers in the same workstation this delay is called unavoidable delay.

The objective is to minimize the total cost per production unit and the decisions involved in cost-oriented MALBP include the followings: (1) first how many workers should be assigned to each station then (2) which tasks to be performed by which worker. The notations used in the mathematical model is presented in Table 1.

Table 1 Notations used in the mathematical model

i, h	Task
j	Station
l	Worker
I	Set of tasks
L	Set of workers
J	Set of workstations
$P_i (P_i^*)$	Set of direct (all) predecessors of task i
$F_i (F_i^*)$	Set of direct (all) successors of task i
C	Cycle time (TU / PU)
m	Number of stations
M	A big positive number
MC	Maximum concentration of workers in a station
N	Number of tasks
d_i^t	Duration of task i when there are k workers in the station (TU)
k^{sc}	Cost of capital per station (MU / PU)
k_{jl}^{sw}	Wage rate of worker l in station j . (MU / TU)
k_i^{tw}	Wage rate of task i . (MU / TU)
$x_{ijl} \in \{0,1\}$	Equals 1 if task i is assigned to worker l in station j .
$y_{ih} \in \{0,1\}$	Equals to 1 if task i and h is assigned to the same worker and task i is performed earlier than task h .
st_i	Start time of task i

The problem under consideration is formulated as follows:

$$\min \left(\sum_{j \in J} \sum_{l \in L} j \times x_{Njl} \right) \times k^{sc} + \left(\sum_{l \in L} \sum_{j \in J} k_{jl}^{sw} \right) \times C \tag{1}$$

$$\sum_{j \in J} \sum_{l \in L} x_{ijl} = 1 \quad \forall i \in I \tag{2}$$

$$\sum_{j \in J} \sum_{l \in L} j \times x_{hjl} \leq \sum_{j \in J} \sum_{l \in L} j \times x_{ijl} \quad \forall i \in I, h \in P_i \tag{3}$$

$$st_i + d_i^t \leq C \quad \forall i \in I, j \in J \tag{4}$$

$$st_i - st_h + M \times \left(1 - \sum_{l \in L} x_{hjl} \right) + M \times \left(1 - \sum_{l \in L} x_{ijl} \right) \geq d_h^t \quad \forall i \in I, h \in P_i, j \in J \tag{5}$$

$$st_h - st_i + M \times (1 - x_{hjl}) + M \times (1 - x_{ijl}) + M \times (1 - y_{ih}) \geq d_i^t \quad \forall i \in I, j \in J, l \in L \tag{6}$$

$$st_i - st_h + M \times (1 - x_{hjl}) + M \times (1 - x_{ijl}) + M \times (y_{ih}) \geq d_h^t \quad \forall i \in I, j \in J, l \in L \tag{7}$$

$$h \in \{r | r \in I - (P_i^* \cup F_i^*) \wedge i < r\}$$

$$\sum_{i \in I} x_{ij,l+1} \leq N \times \sum_{i \in I} x_{ijl}$$

$$k_i^{tw} \times x_{ijl} \leq k_{jl}^{sw}$$

$$st_i \geq 0$$

$$k_{jl}^{sw} \geq 0$$

$$x_{ijl} \in \{0,1\}$$

$$y_{ih} \in \{0,1\}$$

$$\forall j \in J, l \in L \tag{8}$$

$$\forall i \in I, j \in J, l \in L \tag{9}$$

$$\forall i \in I \tag{10}$$

$$\forall j \in J, l \in L \tag{11}$$

$$\forall i \in I, j \in J, l \in L \tag{12}$$

$$\forall i \in I$$

$$h \in \{r \mid r \in I - (P_i^* \cup F_i^*) \wedge i < r\} \tag{13}$$

In this formulation equation (1) indicate the objective function to be minimized which is the total cost per production unit. The first term presents the cost of capital which equals to the number of stations multiplied by k^{sc} (cost of capital per station). It is assumed that the task N is successor of all of the tasks in the precedence graph; if that's not the case a fictitious task with zero duration and wage rate must be considered. Therefore task N is always assigned to the last station. The second term in equation (1) present the costs of labor which is the sum of wage rates of all workers in all stations multiplied by cycle time. Constraints (2) imply that each task i must be assigned to exactly one worker in one station. Constraints (3) ensure that precedence relations are observed. Equations (4) imply that all tasks must be finished before the cycle time. Equations (5) indicate that if task h is a direct predecessor of task i and they both assigned to the same station then starting time of task i must be greater than or equal to the finish time of task h. Constraint pair (6) and (7) is disjunctive for large enough values of M. This means that only one of the is active at the same time. Only when tasks i and h don't have any precedence relation and are both assigned to the same worker in the same station this pair becomes active. If $y_{ih}=0$ equation (6) becomes redundant and equation (7) implies that $st_i \geq st_h + d_h^t$ implying that task i must be scheduled after task h. on the other hand if $y_{ih}=1$ equation (7) becomes redundant and equation (6) implies that $st_h \geq st_i + d_i^t$ indicating that task h must be scheduled after task i. Constraint (8) indicate that in each station, workers are used in an increasing order of their indexes. Equation (9) imply that among all tasks assigned to worker l in station j the maximum wage rate is set to be the wage rate of the worker. Equations (10) and (11) ensure that start times and wage rates are non-negative. Equations (12) and (13) indicate that x_{ijl} and y_{ih} are binary variables.

3. Heuristic algorithms developed

Since the traditional cost oriented assembly line balancing problem is NP-hard [17], [19] and the problem considered here is a generalization of it, the problem considered in this paper is also NP-hard. Therefore it is justified to develop heuristic algorithms to obtain good solutions in a computational time short enough to be applied in industrial real instances.

In this paper seven priority rule based heuristic algorithms are presented to solve the problem under consideration. These rules are as follows:

- Max_D: maximum task duration [25].

- Max_R: maximum ranked positional weight which is $\max\{r_i \mid i \in I\}$ where

$$r_i = \begin{cases} d_i^t + \sum_{j \in F_i} r_j, & \text{if } F_i \neq \emptyset \\ d_i^t & \text{if } F_i = \emptyset \end{cases} \tag{26}, [27].$$

- Max_F: maximum number of immediate followers in the precedence graph [25].
- Max_Kt: maximum cost rate [17].
- Min_Kt: minimum cost rate [16].
- Min_Kts: minimal absolute difference to the workers current cost rate i.e. $\min\{|k_i^t - k_{jl}^{sw}| \mid i \in I\}$. This rule is a modification of the rule proposed by Steffen [16].
- Min_Ki: best change of idle cost i.e.

$\min\{\Delta k_i \mid i \in I\}$; where

$$\Delta k_i = \begin{cases} -d_i^t k_i^t, & \text{if } k_i^t \leq k_{jl}^{sw} \\ (k_i^t - k_{jl}^{sw})C - d_i^t k_i^t, & \text{if } k_i^t > k_{jl}^{sw} \end{cases}$$

This rule is a modification of the rule proposed by Amen [20].

The first five rules are static; this means that the priority of tasks doesn't change throughout building a solution. All of the static rules use a main procedure to assign tasks to workers in each station. This procedure is as follows:
 Step 1: Set the current station $Sc=1$, and available tasks $Avail_task = \{1, 2 \dots N\}$. Available tasks are the tasks that haven't been assigned to any worker in any station.
 Step 2: Set the number of workers in the station $Wn=1$ and number of $Tc=0$.
 Step 3: Among tasks of $Avail_task$, ones that are assignable to station Sc , select the task with the highest priority, according to one of the priority rules which will be presented later in this section. Assign it to the worker that starts the task earlier ties are broken in favor of the worker that is not idle i.e. assigning the task to the worker does not lead to unavoidable idle times. The final tie breaker is the index of the worker and the worker with lower index has more priority than the one with higher index. Delete the task from $Avail_task$ then set $Tc=Tc+1$. If the selected worker is empty then set $Wn=Wn+1$. Repeat this step until there is no task assignable to station Sc . Then go to step 4. A task is assignable to a station if it has no predecessor in $Avail_task$ and assigning it to the station doesn't violate the cycle time constraint.

Step 4: A station W_n number of workers is completed. If $Avail_task$ is empty end the procedure, otherwise set $Sc=Sc+1$ and go to step 2.

The last two rules are dynamic and the priority of tasks may change throughout building a solution. These rules are also dependent on the worker to which the task is assigned. Therefore another procedure is needed to build a solution with these rules. This procedure is as follows:

Step 1: Set the current station $Sc=1$, and available tasks $Avail_task = \{1, 2, \dots, N\}$. Available tasks are the tasks that haven't been assigned to any worker in any station.

Step 2: Set the number of workers in the station $W_n=1$ and number of $Tc=0$.

Step 3: Among tasks of $Avail_task$, ones that are assignable to station Sc , select the task with the highest priority. This involves selecting a pair (i, l) of task i and worker l which has the highest priority. Ties are broken in favor of the pairs that don't create idle times; second level ties are broken in favor of lower worker indexes. Finally the third level ties are broken in favor of lower task indexes. Delete the task from $Avail_task$ then set $Tc=Tc+1$. If the selected worker is empty then set $W_n=W_n+1$. Repeat this step until there is no task assignable to station Sc . Then go to step 4. A task is assignable to a station if it has no predecessor in $Avail_task$ and assigning it to the station doesn't violate the cycle time constraint.

Step 4: A station W_n number of workers is completed. If $Avail_task$ is empty end the procedure, otherwise set $Sc=Sc+1$ and go to step 2.

Therefore five static and two dynamic rules are presented in this section to solve the problem.

Algorithm_{sol} is the solution obtained by a given algorithm on a given instance, LB is the lower bound for the instance. To calculate a lower bound on the total costs lower bounds on the costs of capital and the costs of labor is needed. At first the lower bound on the costs of capital is explained. To calculate a lower bound on the costs of capital a lower bound on the number of stations is needed. It is assumed that the first task in the precedence graph is predecessor of all other tasks. Similarly it is assumed that the last task in the graph is successor of all of the other tasks. If there is no such tasks, fictitious tasks is to be considered. To obtain a lower bound on the number of stations, the longest path, also called critical path, from the first task to the last task is considered. The length of this path is a lower bound on the time needed to produce one commodity, lessening or increasing the

number of workers in each station does not change this value. Thus, the formulation for lower bound is:

$$LB_{station} = \left\lceil \frac{\sum_{j \in \text{critical path}} t_j}{C} \right\rceil \quad (2)$$

Therefore a lower bound on the costs of capital is obtained using the following formulation:

$$LB_{capital\ cost} = LB_{station} \times k^{sc} \quad (3)$$

To obtain a lower bound on the costs of labor at first a lower bound on the number of workers is calculated using the formulation [11]:

$$LB_{worker} = \left\lceil \frac{\sum_{j \in I} t_j}{C} \right\rceil \quad (4)$$

Therefore at least LB_{worker} workers are needed. The lower bound on the costs of labor can be computed using the following formulation:

$$LB_{labor\ cost} = C \times MWR \quad (5)$$

In this formulation MWR is the sum of LB_{worker} smallest wage rate values. Therefore the lower bound on the costs of production is computed using the following formula:

$$LB = LB_{capital\ cost} + LB_{labor\ cost} \quad (6)$$

4. Computational results

In this section computational experiments are presented. In the first experiment the motivation is comparing the cost-oriented model with the traditional time-oriented model. Therefore an example is presented and solved with both time-oriented and cost-oriented approaches. The precedence graph and task times for this example are taken from the well-known instance of Mertens which is available at www.assembly-line-balancing.de. In the time oriented model at first the number of workers is minimized as the primary objective and then the number of stations is minimized as the secondary objective. The precedence graph of this example with duration and wage rate of tasks is shown in fig.4. The cycle time and maximum feasible worker concentration in each station for this instance is assumed to be 8 and 3 respectively. Also total cost of capital per station (k^{sc}) is assumed to be equal to 5.

Optimum solutions for cost-oriented and time-oriented versions of this problem are presented in figures 5 and 6 respectively. In these figures for each task, starting time and finishing time are shown alongside its bar. Shaded rectangles designate idle time at the end of the cycle time.

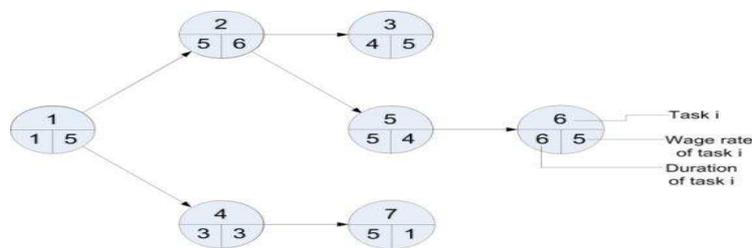


Fig. 1 example of a problem instance

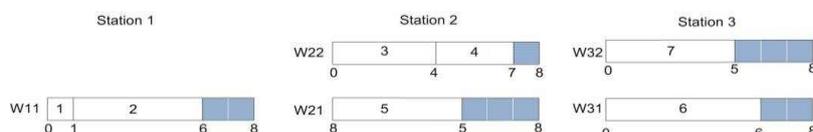


Fig. 2 The optimum solution for cost-oriented approach

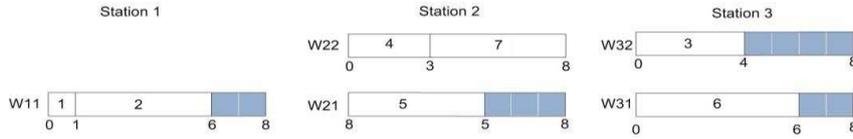


Fig. 3 The optimum solution for time-oriented approach

Table 2 shows the cost calculations for the optimal solutions obtained by time-oriented and cost-oriented approaches. As seen from this table a total of 199 monetary units are needed to produce one production unit are being used in the traditional line balancing model, while this number could be reduced to 183 with the proposed model. Thus, 16 monetary units are saved. Besides, the required number of workers and

stations are calculated as 5 and 3 respectively. These numbers are the same with the ones obtained by the time-oriented model, which means that the solution is also optimal in terms of number of workers and stations. Consequently, the solution is the best in terms of the total cost, and while reaching this best, the best number of stations and workers are also achieved.

Table 2 Optimal solutions to the example

Station	Worker	Time-oriented optimal solution		Cost-oriented optimal solution	
		I_{ji}^s	k_{ji}^{sw}	I_{ji}^s	k_{ji}^{sw}
1	1	{1,2}	6	{1,2}	6
2	1	{5}	4	{5}	4
	2	{4,7}	3	{3,4}	5
3	1	{6}	5	{6}	5
	2	{3}	5	{7}	1
$\sum_{i \in L} \sum_{j \in J} k_{ji}^{sw}$			23		21
m			3		3
$k = \sum_{i \in L} \sum_{j \in J} C \times k_{ji}^{sw} + m \times k^{sc}$			199		183

In the second experiment the act of the suggested algorithms is illustrated. To do so, each of the 25 different precedence graphs available at www.assembly-line-balancing.de is used to generate an instance. For each task in each instance the wage rate of task i is assumed to be: $k_i^{fw} = d_{N+1-i}^t$. The cycle time is generated randomly between maximum task time t_{max} and $2 * t_{max}$. The cost of capital for each station is assumed to be: $k^{sc} = \frac{c^2}{2}$. Each instance is solved by the proposed algorithms and the relative deviation is computed using equation (1) in section 3.6. The results are presented in fig. 7.

As seen in fig. 7 the two dynamic priority rules, Min_Kts and Min_Ki, have a better overall performance comparing to other priority rules. This highlights the importance of considering the current cost rate of the workers while building the solution. For this experiment another data set is generated using a selection of well-known instances for SALBP-1. In order to facilitate comparison of the proposed algorithm with other future algorithms, the wage rates for each task: $k_i^{fw} = d_{N+1-i}^t$ and cost of capital for each station is set to be: $k^{sc} = \frac{c^2}{2}$.

In this equation f is the space utilization factor, m is the number of stations and tw is the number of workers. This factor ranges between 1 and $1/tw$ and is of special importance if there are space constraints in the production floor which may happen because of the building design or redesigning the line to produce a new product.

The multi-manned system results in a shorter physical line length and improves the space utilization. Because in this

system the same number of workers can be allocate to fewer stations comparing to the traditional approach. In Table 2, in many instances the space utilization factor has improved and for all examples the average space utilization factor is 45.95 percent. This means that the required space has reduced to 45.95 percent of its previous value for the traditional approach.

5. Conclusions and future research

MALs are a new type of lines in which there can be more than one worker in each station working on the same work-piece. This type of line is very common in manufacturing of large-sized products e.g. vehicle final assembly. MALs have several advantages over the traditional lines which include reducing the length of the line and better utilization of the tools and machinery in stations. On the other hand this type of lines results in reducing the work in process and throughput time which is of high priority for production managers.

In the classical MALBP the objective is to minimize the manpower needed to manufacture one product unit. Apart from the manpower, other cost drivers like wage rates or machinery are neglected in this classical view of the problem. But due to the high competition in the current production environment, reducing the production costs and increasing utilization of available resources are very important issues for manufacturing managers.

Although minimizing the costs of production is of major importance in practice, there has not been sufficient consideration in the literature of MAL. In this paper the

MALBP is considered with the aim of minimizing the total costs of one production unit. For this aim a mathematical formulation is presented. Furthermore in order to be able to solve the medium- and large-size scales of the problem, several heuristics are proposed. Several examples are solved to show the effectiveness of the proposed model and proposed algorithms.

However, since the tasks are performed by human being, it is reasonable to assume task times be stochastic. Therefore the current research can be extended to the stochastic environments in MALs and incompleteness costs can be additionally considered. Also developing other heuristic or meta-heuristics such as Tabu search or ant colony optimization to solve the introduced model is recommended for future research in this area.

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