European Journal of Academic Essays 1(2): 6-16, 2014
ISSN: 2183-1904
www.euroessays.org

Exact Traveling Wave Solutions for Coupled Nonlinear Fractional pdes Using the (G'/G)-Expansion Method

Elsayed M. E. Zayed [1], Ahmed H. Arnous [2]

[1]Mathematics Department, Faculty of Science, Zagazig University,
Zagazig, Egypt
e.m.e.zayed@hotmail.com

[2]Engineering Mathematics and Physics Department, Higher Institute of Engineering,
El Shorouk, Egypt
ahmed.h.arnous@gmail.com

Abstract: In this paper, the (G'/G) -expansion method is extended to solve fractional differential equations in the sense of modified Riemann-Liouville derivative. Based on a nonlinear fractional complex transformation, certain fractional partial differential equations can be turned into ordinary differential equations of integer order. For illustrating the validity of this method, we apply it to find exact solutions with parameters for four fractional nonlinear partial differential equations namely, the time fractional nonlinear coupled Burgers equations, the time fractional nonlinear coupled KdV equations, the time fractional nonlinear Zoomeron equation and the time fractional nonlinear Klein-Gordon-Zakharov equations . When these parameters are taken to be special values, the solitary wave solutions are derived from the exact solutions. The proposed method is efficient and powerful in solving wide classes of nonlinear evolution fractional order equations.

Keywords: The (G'/G)-expansion method, Fractional partial differential equations, Modified Riemann-Liouville derivative, Exact solutions; Solitary wave solutions, Fractional complex transformation.

1. Introduction

During the past decades, more and more nonlinear fractional differential equations in mathematical physics are playing a major role in various fields. These equations appear in a wide great array of contexts, such as physics, biology, electromagnetic, electrochemistry, engineering and fractional dynamics [1, 2]. Consequently, considerable attention has been given to the solutions of these equations. Many powerful methods for solving nonlinear fractional order partial differential equations were appeared in open literature, such as Bäcklund transformation [3], the homotopy perturbation method [4,5], the exp-functional method [6], the (G'/G)-expansion method [7], the improved (G'/G)-expansion method [8], the Adomian decomposition method [9,10] and so on. Finding the exact solutions to fractional differential equations is an important task. It is therefore needed to find a proper method to solve the problem of nonlinear differential equations containing fractional calculus. In the present article, based on the homogeneous balance principle and Jumarie's modified Riemann–Liouville derivative [11-15], we will apply the (G'/G)-expansion method for solving the nonlinear fractional PDEs in the sense of the modified Riemann-Liouville derivative obtained by Jumarie. It is well-known that the modified Riemann-Liouville derivative of order α is defined by the following expression:

$$D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dt} \displaystyle\int_0^t (t-\eta)^{-\alpha} [f(\eta) - f(0)] d\eta, \\ \left[f^{(n)}(t) \right]^{\alpha - n}, \ n \le \alpha < n+1, \ n \ge 1. \end{cases} \quad (1)$$

We list some important properties for the modified Riemann-Liouville derivative as follows:

$$\begin{aligned} &D_t^{\alpha} t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)} t^{r-\alpha}, \ r > 0, \\ &D_t^{\alpha} \left[f(t) g(t) \right] = f(t) D_t^{\alpha} g(t) + g(t) D_t^{\alpha} f(t), \\ &D_t^{\alpha} \left[f(g(t)) \right] = f_g'(g(t)) D_t^{\alpha} g(t) = D_g^{\alpha} f(g(t)) [g_t'(t)]^{\alpha}. \end{aligned} \quad (2)$$

Corresponding Author: Elsayed M. E. Zayed, Email: e.m.e.zayed@hotmail.com

2. Description of the (G'/G) -expansion method for fractional partial differential equations

Suppose that a fractional partial differential equation, say in the independent variables $t, x_1, ..., x_n$ is given by

$$F\left(u, D_t^{\alpha}u, u_{x_1}, ..., u_{x_n}\right) = 0, \tag{3}$$

where $u = u(t, x_1, ..., x_n)$ are unknown functions, $0 < \alpha \le 1$, F is a polynomial of the unknown function $u = u(t, x_1, ..., x_n)$ and its partial fractional derivatives, in which the highest order fractional derivatives and the nonlinear terms are involved. These derivatives denote Jumarie's fractional derivatives, which are the modified Riemann-Liouville derivatives . In the following, we give the main steps of the method:

Step 1. Li et al. [14,15] have proposed the fractional complex transform to convert fractional order partial differential equations with the modified Riemann-Liouville fractional derivative into integer order ordinary differential equations. Thus, the fractional complex transform is a natural extension of the traveling wave transform. So, we assume that

$$u(x, y, t) = U(\xi), \tag{4}$$

where

$$\xi = k\, x + \frac{\omega t^{\alpha}}{\Gamma(1+\alpha)}, \tag{5}$$

where k, ω are nonzero constants, to reduce Eq. (3) to the following integer order ordinary differential equation:

$$P(U, U', U'', ...) = 0, \tag{6}$$

where P is a polynomial in $u(\xi)$ and its total derivatives, while $' = \dfrac{d}{d\xi}$.

Step 2. We suppose that Eq. (3) has the formal solution

$$U(\xi) = \sum_{j=0}^{N} A_j \left(\frac{G'(\xi)}{G(\xi)}\right)^j, \tag{7}$$

Where A_j are constants to be determined, such $A_N \neq 0.$ the function $G(\xi)$ satisfies the second-order linear ODE in the form

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{8}$$

Which have the following solutions

$$\left(\frac{G'}{G}\right) = \begin{cases} \dfrac{-\lambda}{2} + \dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\left(\dfrac{c_1 \sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right) + c_2 \cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1 \cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right) + c_2 \sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right) \\ \dfrac{-\lambda}{2} + \dfrac{\sqrt{4\mu - \lambda^2}}{2}\left(\dfrac{-c_1 \sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right) + c_2 \cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1 \cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right) + c_2 \sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right) \\ \dfrac{-\lambda}{2} + \dfrac{c_2}{c_1 + c_2 \xi}, \end{cases} \tag{9}$$

Where c_1, c_2, μ, λ are arbitrary constants.

Step3. We determine the positive integer in (7) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (6).

Step4. We substitute (7) along with Eq. (8) into Eq. (6) and collect all terms with the same powers of (G'/G) and setting them to zero, we have a system of algebraic equations, which can be solved using the Maple or Mathematica. Consequently, we can obtain the exact solutions of Eq. (3).

3. Applications

In this section, we will use the proposed method of Sec. 2, to construct the exact solutions of the following fractional nonlinear partial differential equations:

3.1. Example 1: The time fractional nonlinear coupled Burgers equations

In this subsection, we consider the following time fractional nonlinear coupled Burgers equations [16, 17]:

$$\begin{cases} D_t^{\alpha}u - u_{xx} - \left(u^2\right)_x + \beta\left(uv\right)_x = 0, \\ D_t^{\alpha}v - v_{xx} - \left(v^2\right)_x + \gamma\left(uv\right)_x = 0, \ 0 < \alpha \le 1, \end{cases} \tag{10}$$

Where β, γ are nonzero different constants. In order to solve the system (10), we use the fractional complex transformations

$$u(x,t) = U(\xi),\ v(x,t) = V(\xi),\ \xi = k\, x + \frac{\omega t^{\alpha}}{\Gamma(1+\alpha)}, \tag{11}$$

Where k, ω are nonzero constants, to reduce the system (10) to the following ordinary differential equations:

$$\begin{cases} \omega U' - k^2 U'' - k\left(U^2\right)' + \beta\left(UV\right)' = 0, \\ \omega V' - k^2 V'' - k\left(V^2\right)' + \gamma\left(UV\right)' = 0, \ 0 < \alpha \le 1 \end{cases} \tag{12}$$

Integrating the system (12) with zero constant of integration, we have

$$\begin{cases}\omega U - k^2 U' - k\left(U^2\right) + \beta\left(UV\right) = 0, \\ \omega V - k^2 V' - k\left(V^2\right) + \gamma\left(UV\right) = 0, \quad 0 < \alpha \le 1\end{cases} \tag{13}$$

Balancing U' with U^2 and V' with V^2 with in the system (13), we get

$$U(\xi) = a_0 + a_1\left(\frac{G'}{G}\right), \tag{14}$$

and

$$V(\xi) = b_0 + b_1\left(\frac{G'}{G}\right), \tag{15}$$

where a_0, a_1, b_0 and b_1 are constants to be determined later, such that $a_1 \neq 0$ and $b_1 \neq 0$. Substituting Eqs. (14) ,(15) and their derivatives into Eq.(13) and collecting all terms with the same powers of $(G'/G)^i, i = 0,1,2$ together and equating each coefficient to zero, we have the following system of algebraic equations:

$$0:\ \omega a_0 - k a_0^2 + k^2 \mu a_1 + k\beta a_0 b_0 = 0,$$
$$0:\ \omega b_0 + k\gamma a_0 b_0 - k b_0^2 + k^2 \mu b_1 = 0,$$
$$1: k^2 \lambda a_1 + \omega a_1 - 2k a_0 a_1 + k\beta a_1 b_0 + k\beta a_0 b_1 = 0,$$
$$1: k\gamma a_1 b_0 + k^2 \lambda b_1 + \omega b_1 + k\gamma a_0 b_1 - 2k b_0 b_1 = 0,$$
$$2: k^2 a_1 - k a_1^2 + k\beta a_1 b_1 = 0,$$
$$2: k^2 b_1 + k\gamma a_1 b_1 - k b_1^2 = 0.$$

Solving these algebraic equations using the Maple or Mathematical yields

$$a_0 = \frac{(1+\beta)b_0}{1+\gamma},\ a_1 = \frac{k(1+\beta)}{1-\beta\gamma},\ b_1 = \frac{k(1+\gamma)}{1-\beta\gamma},$$
$$\lambda = -\frac{(1+\gamma)\omega + 2k(\beta\gamma-1)b_0}{k^2(1+\gamma)},$$
$$\mu = \frac{b_0(\beta\gamma-1)\left[(1+\gamma)\omega + (\beta\gamma-1)kb_0\right]}{k^3(1+\gamma)^2}.$$

Now we get the following exact traveling wave solutions:

(1) If $\lambda^2 - 4\mu > 0,$ then we have

$$\begin{cases} U(\xi) = \dfrac{(1+\beta)b_0}{1+\gamma} - \dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)} + \dfrac{k(1+\beta)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)} \times \\ \left(\dfrac{c_1 \sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right) + c_2 \cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1 \cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right) + c_2 \sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right), \\ V(\xi) = b_0 - \dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)} + \dfrac{k(1+\gamma)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)} \times \\ \left(\dfrac{c_1 \sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right) + c_2 \cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1 \cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right) + c_2 \sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right), \end{cases} \tag{16}$$

Substituting Eqs. (8), (10), (12), (14) obtained by Peng [18] into Eq.(16), we have respectively the following Kink-type traveling wave solutions:

(i) If $|c_1| > |c_2|$

$$\begin{cases} U(\xi) = \dfrac{(1+\beta)b_0}{1+\gamma} - \dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)} + \dfrac{k(1+\beta)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)} \times \\ \tanh\left[\dfrac{\xi}{2}\sqrt{\lambda^2-4\mu} + \operatorname{sgn}(c_1c_2)\psi_1\right], \\ V(\xi) = b_0 - \dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)} + \dfrac{k(1+\gamma)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)} \times \\ \tanh\left[\dfrac{\xi}{2}\sqrt{\lambda^2-4\mu} + \operatorname{sgn}(c_1c_2)\psi_1\right], \end{cases} \tag{17}$$

(ii) If $|c_2| > |c_1| \neq 0,$

$$\begin{cases} U(\xi) = \dfrac{(1+\beta)b_0}{1+\gamma} - \dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)} + \dfrac{k(1+\beta)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)} \times \\ \coth\left[\dfrac{\xi}{2}\sqrt{\lambda^2-4\mu} + \operatorname{sgn}(c_1c_2)\psi_2\right], \\ V(\xi) = b_0 - \dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)} + \dfrac{k(1+\gamma)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)} \times \\ \coth\left[\dfrac{\xi}{2}\sqrt{\lambda^2-4\mu} + \operatorname{sgn}(c_1c_2)\psi_2\right], \end{cases} \tag{18}$$

where $\psi_1 = \tanh^{-1}\left(\dfrac{|c_2|}{|c_1|}\right),\ \psi_2 = \coth^{-1}\left(\dfrac{|c_2|}{|c_1|}\right),$ and $\operatorname{sgn}(c_1c_2)$ is the sign function.

(iii) If $|c_2| > |c_1| = 0,$

$$\begin{cases} U(\xi)=\dfrac{(1+\beta)b_0}{1+\gamma}-\dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)}+ \\ \dfrac{k(1+\beta)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)}\coth\left[\dfrac{\xi}{2}\sqrt{\lambda^2-4\mu}\right], \\ V(\xi)=b_0-\dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)}+ \\ \dfrac{k(1+\gamma)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)}\coth\left[\dfrac{\xi}{2}\sqrt{\lambda^2-4\mu}\right], \end{cases} \quad (19)$$

(iv) If $|c_2|=|c_1|$,

$$\begin{cases} U(\xi)=\dfrac{(1+\beta)b_0}{1+\gamma}-\dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)}+\dfrac{k(1+\beta)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)}, \\ V(\xi)=b_0-\dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)}+\dfrac{k(1+\gamma)\sqrt{\lambda^2-4\mu}}{2(1-\beta\gamma)}, \end{cases} \quad (20)$$

(2) If $\lambda^2-4\mu<0,$ then we get

$$\begin{cases} U(\xi)=\dfrac{(1+\beta)b_0}{1+\gamma}-\dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)}+\dfrac{k(1+\beta)\sqrt{4\mu-\lambda^2}}{2(1-\beta\gamma)}\times \\ \left(\dfrac{-c_1\sin\left(\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\cos\left(\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1\cos\left(\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\sin\left(\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right), \\ V(\xi)=b_0-\dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)}+\dfrac{k(1+\gamma)\sqrt{4\mu-\lambda^2}}{2(1-\beta\gamma)}\times \\ \left(\dfrac{-c_1\sin\left(\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\cos\left(\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1\cos\left(\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\sin\left(\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right). \end{cases} \quad (21)$$

Now, we simplify Eq. (21) to get the following periodic solutions:

$$\begin{cases} U(\xi)=\dfrac{(1+\beta)b_0}{1+\gamma}-\dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)}+ \\ \dfrac{k(1+\beta)\sqrt{4\mu-\lambda^2}}{2(1-\beta\gamma)}\tan\left[\xi_1-\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right], \\ V(\xi)=b_0-\dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)}+ \\ \dfrac{k(1+\gamma)\sqrt{4\mu-\lambda^2}}{2(1-\beta\gamma)}\tan\left[\xi_1-\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right], \end{cases} \quad (22)$$

$$\begin{cases} U(\xi)=\dfrac{(1+\beta)b_0}{1+\gamma}-\dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)}+ \\ \dfrac{k(1+\beta)\sqrt{4\mu-\lambda^2}}{2(1-\beta\gamma)}\tan\left[\xi_2+\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right], \\ V(\xi)=b_0-\dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)}+ \\ \dfrac{k(1+\gamma)\sqrt{4\mu-\lambda^2}}{2(1-\beta\gamma)}\tan\left[\xi_2+\dfrac{\xi}{2}\sqrt{4\mu-\lambda^2}\right], \end{cases} \quad (23)$$

where $\xi_1=\tan^{-1}\dfrac{c_2}{c_1},\ \xi_2=\cot^{-1}\dfrac{c_2}{c_1}.$

(3) If $\lambda^2-4\mu=0,$

$$\begin{cases} U(\xi)=\dfrac{(1+\beta)b_0}{1+\gamma}-\dfrac{k\lambda(1+\beta)}{2(1-\beta\gamma)}+\dfrac{k(1+\beta)c_2}{(1-\beta\gamma)(c_1+c_2\xi)}, \\ V(\xi)=b_0-\dfrac{k\lambda(1+\gamma)}{2(1-\beta\gamma)}+\dfrac{k(1+\gamma)c_2}{(1-\beta\gamma)(c_1+c_2\xi)}, \end{cases} \quad (24)$$

3.2. Example 2: The time fractional nonlinear coupled KdV equations

We considered the following time fractional nonlinear coupled KdV equations [19]:

$$\begin{cases} D_t^{\alpha}u+6auu_x-2bvv_x+au_{xxx}=0, \\ D_t^{\alpha}u+3buv_x+bv_{xxx}=0,\ 0<\alpha\le 1, \end{cases} \quad (25)$$

Where a and b are nonzero constants. The transformations (11) reduce Eqs. (25) to the ODEs:

$$\begin{cases} \omega U'+6akUU'-2bkVV'+ak^3U'''=0, \\ \omega U'+3bkUV'+bk^3V'''=0, \end{cases} \quad (26)$$

Balancing U''' with VV' and V''' with UV' in the system (26) we get

$$U(\xi)=a_0+a_1\left(\frac{G'}{G}\right)+a_2\left(\frac{G'}{G}\right)^2, \quad (27)$$

and

$$V(\xi)=b_0+b_1\left(\frac{G'}{G}\right)+b_2\left(\frac{G'}{G}\right)^2, \quad (28)$$

Where a_0,a_1,b_0,b_1,a_2 and b_2 are constants to be determined later, such that $a_2\neq 0$ and $b_2\neq 0$. Substituting Eqs.(27) and (28) into Eq.(26) and collecting all terms with the same powers of $(G'/G)^i$, $i=0,1,2,3,4,5$ together and equating each coefficient to zero, we have the following system of algebraic equations:

$$0:-ak^3\lambda^2\mu a_1-2ak^3\mu^2a_1-\mu\omega a_1-6ak\mu a_0a_1-6ak^3\lambda\mu^2a_2+2bk\mu b_0b_1=0,$$
$$0:-\mu\omega a_1-bk^3\lambda^2\mu b_1-2bk^3\mu^2b_1-3bk\mu a_0b_1-6bk^3\lambda\mu^2b_2=0,$$
$$1:-ak^3\lambda^3a_1-8ak^3\lambda\mu a_1-\lambda\omega a_1-6ak\lambda a_0a_1-6ak\mu a_1^2-14ak^3\lambda^2\mu a_2-16ak^3\mu^2a_2-2\mu\omega a_2-12ak\mu a_0a_2+2bk\lambda b_0b_1+2bk\mu b_1^2+4bk\mu b_0b_2=0,$$
$$1:-\lambda\omega a_1-2\mu\omega a_2-bk^3\lambda^3b_1-8bk^3\lambda\mu b_1-3bk\lambda a_0b_1-3bk\mu a_1b_1-14bk^3\lambda^2\mu b_2-16bk^3\mu^2b_2-6bk\mu a_0b_2=0,$$
$$2:-7ak^3\lambda^2a_1-8ak^3\mu a_1-\omega a_1-6aka_0a_1-6ak\lambda a_1^2-8ak^3\lambda^3a_2-52ak^3\lambda\mu a_2-2\lambda\omega a_2-12ak\lambda a_0a_2-18ak\mu a_1a_2+2bkb_0b_1+2bk\lambda b_1^2+4bk\lambda b_0b_2+6bk\mu b_1b_2=0,$$
$$2:-\omega a_1-2\lambda\omega a_2-7bk^3\lambda^2b_1-8bk^3\mu b_1-3bka_0b_1-3bk\lambda a_1b_1-3bk\mu a_2b_1-8bk^3\lambda^3b_2-52bk^3\lambda\mu b_2-6bk\lambda a_0b_2-6bk\mu a_1b_2=0,$$
$$3:-12ak^3\lambda a_1-6aka_1^2-38ak^3\lambda^2a_2-40ak^3\mu a_2-2\omega a_2-12aka_0a_2-18ak\lambda a_1a_2-12ak\mu a_2^2+2bkb_1^2+4bkb_0b_2+6bk\lambda b_1b_2+4bk\mu b_2^2=0,$$
$$3:-2\omega a_2-12bk^3\lambda b_1-3bka_1b_1-3bk\lambda a_2b_1-38bk^3\lambda^2b_2-40bk^3\mu b_2-6bka_0b_2-6bk\lambda a_1b_2-6bk\mu a_2b_2=0,$$
$$4:-6ak^3a_1-54ak^3\lambda a_2-18aka_1a_2-12ak\lambda a_2^2+6bkb_1b_2+4bk\lambda b_2^2=0,$$
$$4:-6bk^3b_1-3bka_2b_1-54bk^3\lambda b_2-6bka_1b_2-6bk\lambda a_2b_2=0,$$
$$5:-24ak^3a_2-12aka_2^2+4bkb_2^2=0,$$
$$5:-24bk^3b_2-6bka_2b_2=0.$$

Solving these algebraic equations by Maple or Mathematica yields

Case 1:

$$b_0=-\frac{6a_0(3k^2\lambda+b_1)+k^2(\lambda^2+8\mu)(6k^2\lambda+b_1)}{12k^2\lambda},$$
$$\omega=\frac{b(k^2(\lambda^2+8\mu)+3a_0)b_1}{4k\lambda},b_2=\frac{b_1}{\lambda},$$
$$a_1=-4k^2\lambda,\ a_2=-4k^2,\ b_1\neq 0,$$

Case 2:

$$b_0=-\frac{6a_0(3k^2+b_2)+k^2\lambda^2(6k^2+b_2)}{12k^2},$$
$$\omega=\frac{b(k^2\lambda^2+3a_0)b_2}{4k},b_1=\lambda b_2,a_1=-4k^2\lambda,$$
$$a_2=-4k^2,\ \mu=0,\ b_1\neq 0,$$

Case 3:

$$b_0=\frac{(\sqrt{6a}-3\sqrt{b})k^2\lambda^2+(6\sqrt{6a}-9\sqrt{b})a_0}{6\sqrt{b}},$$
$$\omega=-\sqrt{\frac{3ab}{2}}k(k^2\lambda^2+3a_0),a_1=-4k^2\lambda,$$
$$b_1=-\frac{2\sqrt{6a}k^2\lambda}{\sqrt{b}},\ b_2=-\frac{2\sqrt{6a}k^2}{\sqrt{b}},$$
$$a_2=-4k^2,\ \mu=0,\ b_1\neq 0.$$

For case 1, we have the following results:

(1) If $\lambda^2-4\mu>0$, then

$$\begin{cases}U(\xi)=a_0+k^2\lambda^2-k^2(\lambda^2-4\mu)\times\left(\dfrac{c_1\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right)^2,\\ V(\xi)=b_0-\dfrac{b_1\lambda}{4}+\dfrac{b_1(\lambda^2-4\mu)}{4\lambda}\times\left(\dfrac{c_1\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right)^2.\end{cases}\quad(29)$$

Setting $\phi=\frac{\xi}{2}\sqrt{\lambda^2-4\mu}$ and substituting Eqs.(8), (10), (12), (14) obtained by Peng [18] into Eq.(29), we have respectively the following Kink-type traveling wave solutions:

(i) If $|c_1|>|c_2|$

$$\begin{cases} U(\xi)=a_0+k^2\lambda^2\operatorname{sec}h^2[\phi+\operatorname{sgn}(c_1c_2)\psi_1]+\\ 4\mu k^2\tanh^2[\phi+\operatorname{sgn}(c_1c_2)\psi_1],\\ V(\xi)=b_0-\dfrac{b_1\lambda}{4}\operatorname{sec}h^2[\phi+\operatorname{sgn}(c_1c_2)\psi_1]-\\ \dfrac{\mu b_1}{\lambda}\tanh^2[\phi+\operatorname{sgn}(c_1c_2)\psi_1], \end{cases} \quad (30)$$

(ii) If $|c_2|>|c_1|\neq 0$

$$\begin{cases} U(\xi)=a_0-k^2\lambda^2\operatorname{csc}h^2[\phi+\operatorname{sgn}(c_1c_2)\psi_2]+\\ 4\mu k^2\coth^2[\phi+\operatorname{sgn}(c_1c_2)\psi_2],\\ V(\xi)=b_0+\dfrac{b_1\lambda}{4}\operatorname{csc}h^2[\phi+\operatorname{sgn}(c_1c_2)\psi_2]-\\ \dfrac{\mu b_1}{\lambda}\coth^2[\phi+\operatorname{sgn}(c_1c_2)\psi_2], \end{cases} \quad (31)$$

(iii) If $|c_2|>|c_1|=0$

$$\begin{cases} U(\xi)=a_0-k^2\lambda^2\operatorname{csc}h^2[\phi]+4\mu k^2\coth^2[\phi],\\ V(\xi)=b_0+\dfrac{b_1\lambda}{4}cs\,ch^2[\phi]-\dfrac{\mu b_1}{\lambda}\coth^2[\phi], \end{cases} \quad (32)$$

(iv) If $|c_2|=|c_1|$

$$\begin{cases} U(\xi)=a_0+4\mu,\\ V(\xi)=b_0-\dfrac{b_1\mu}{\lambda}, \end{cases} \quad (33)$$

where $\psi_1=\tanh^{-1}\left(\dfrac{|c_2|}{|c_1|}\right)$, $\psi_2=\coth^{-1}\left(\dfrac{|c_2|}{|c_1|}\right)$, and $\operatorname{sgn}(c_1c_2)$ is the sign function.

(2) If $\lambda^2-4\mu<0$ then

$$\begin{cases} U(\xi)=a_0+k^2\lambda^2-k^2(4\mu-\lambda^2)\times\\ \left(\dfrac{-c_1\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right)^2,\\ V(\xi)=b_0-\dfrac{b_1\lambda}{4}+\dfrac{b_1(4\mu-\lambda^2)}{4\lambda}\times\\ \left(\dfrac{-c_1\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right)^2. \end{cases} \quad (34)$$

Now, we simplify Eq. (34) to get the following periodic solutions:

$$\begin{cases} U(\xi)=a_0+k^2\lambda^2\sec^2[\xi_1-\phi]+\\ 4\mu k^2\tan^2[\xi_1-\phi],\\ V(\xi)=b_0-\dfrac{b_1\lambda}{4}\sec^2[\xi_1-\phi]-\\ \dfrac{\mu b_1}{\lambda}\tan^2[\xi_1-\phi], \end{cases} \quad (35)$$

$$\begin{cases} U(\xi)=a_0+k^2\lambda^2\csc^2[\xi_2+\phi]+\\ 4\mu k^2\cot^2[\xi_2+\phi],\\ V(\xi)=b_0-\dfrac{b_1\lambda}{4}\csc^2[\xi_2+\phi]-\\ \dfrac{\mu b_1}{\lambda}\cot^2[\xi_2+\phi], \end{cases} \quad (36)$$

where $\xi_1=\tan^{-1}\dfrac{c_2}{c_1}$, $\xi_2=\cot^{-1}\dfrac{c_2}{c_1}$.

(3) If $\lambda^2-4\mu=0$, then the rational solution

$$\begin{cases} U(\xi)=a_0+k^2\lambda^2-\dfrac{4k^2c_2^2}{(c_1+c_2\xi)^2},\\ V(\xi)=b_0-\dfrac{b_1\lambda}{4}+\dfrac{b_1c_2^2}{\lambda(c_1+c_2\xi)^2}, \end{cases} \quad (37)$$

where

$$\xi=k\,x+\frac{b(k^2(\lambda^2+8\mu)+3a_0)b_1}{4k\lambda}\frac{t^\alpha}{\Gamma(1+\alpha)}.$$

3.3. Example 3: The time fractional nonlinear Zoomeron equation

We consider the following time fractional nonlinear Zoomeron equation [20]

$$D_t^{2\alpha}\left(\frac{u_{xy}}{u}\right)-\left(\frac{u_{xy}}{u}\right)_{xx}+2D_t^{\alpha}\left(u^2\right)_x=0, \quad (38)$$

In order to solve Eq. (38) , we introduce the following transformation

$$u(x,y,t)=U(\xi),\ \ \xi=k_1x+k_2y+\frac{\omega t^\alpha}{\Gamma(1+\alpha)}, \quad (39)$$

Where k_1,k_2,ω are nonzero constants. Substituting (39) into (38), we have the ODE:

$$k_1k_2\omega^2\left(\frac{U''}{U}\right)''-k_1^3k_2\left(\frac{U''}{U}\right)''+2k_1\omega\left(U^2\right)''=0, \quad (40)$$

Integrating Eq. (40) twice with respect to ξ , we get

$$k_1 k_2\left(\omega^2-k^2\right)U''+2k_1\omega U^3+CU=0, \quad (41)$$

where C is a constant of integration, while the second constant of integration is vanishing. Balancing U'' with U^3 in the Eq. (41), we get

$$U(\xi)=a_0+a_1\left(\frac{G'}{G}\right), \quad (42)$$

Where a_0, a_1 are constants to be determined later, such that $a_1 \neq 0$. Substituting Eq.(42) into Eq.(41) and collecting all terms with the same powers of $(G'/G)^i, i=0,1,2,3$ together and equating each coefficient to zero, we have the following system of algebraic equations:

$$0: Ca_0+2\omega a_0^3 k_1+\lambda\mu\omega^2 a_1 k_1 k_2-\lambda\mu a_1 k_1^3 k_2=0,$$
$$1: Ca_1+6\omega a_0^2 a_1 k_1+\lambda^2\omega^2 a_1 k_1 k_2+2\mu\omega^2 a_1 k_1 k_2-\lambda^2 a_1 k_1^3 k_2-2\mu a_1 k_1^3 k_2=0,$$
$$2: 6\omega a_0 a_1^2 k_1+3\lambda\omega^2 a_1 k_1 k_2-3\lambda a_1 k_1^3 k_2=0,$$
$$3: 2\omega a_1^3 k_1+2\omega^2 a_1 k_1 k_2-2a_1 k_1^3 k_2=0,$$

Solving these algebraic equations using Maple or Mathematica yields

$$a_0=\frac{\lambda a_1}{2},\ a_1=\pm\sqrt{\frac{k_2(k_1^2-\omega^2)}{\omega}},$$
$$\mu=\frac{\lambda^2 k_1 k_2(\omega^2-k_1^2)-2C}{4k_1 k_2(\omega^2-k_1^2)}.$$

(1) If $\lambda^2-4\mu>0,$ then we have

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(\lambda^2-4\mu)}{\omega}}\left(\frac{c_1\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)+c_2\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right) \quad (43)$$

Substituting Eqs.(8), (10), (12), (14) obtained by Peng [18] into Eq.(43), we have respectively the following Kink-type traveling wave solutions:

(i) If $|c_1|>|c_2|,$

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(\lambda^2-4\mu)}{\omega}}\times\tanh\left[\frac{\xi}{2}\sqrt{\lambda^2-4\mu}+\operatorname{sgn}(c_1c_2)\psi_1\right], \quad (44)$$

(ii) If $|c_2|>|c_1|\neq 0,$

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(\lambda^2-4\mu)}{\omega}}\times\coth\left[\frac{\xi}{2}\sqrt{\lambda^2-4\mu}+\operatorname{sgn}(c_1c_2)\psi_2\right], \quad (45)$$

Where $\psi_1=\tanh^{-1}\left(\frac{|c_2|}{|c_1|}\right),\ \psi_2=\coth^{-1}\left(\frac{|c_2|}{|c_1|}\right),$ and $\operatorname{sgn}(c_1c_2)$ is the sign function.

(iii) If $|c_2|>|c_1|=0,$

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(\lambda^2-4\mu)}{\omega}}\coth\left[\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right], \quad (46)$$

(iv) If $|c_2|=|c_1|,$

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(\lambda^2-4\mu)}{\omega}}, \quad (47)$$

(2) If $\lambda^2-4\mu<0,$ then we get

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(4\mu-\lambda^2)}{\omega}}\times\left(\frac{-c_1\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right) \quad (48)$$

Now, we simplify Eq. (48) to get the following periodic solutions:

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(4\mu-\lambda^2)}{\omega}}\tan\left[\xi_1-\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right], \quad (49)$$

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(4\mu-\lambda^2)}{\omega}}\cot\left[\xi_2+\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right], \quad (50)$$

Where $\xi_1=\tan^{-1}\frac{c_2}{c_1},\ \xi_2=\cot^{-1}\frac{c_2}{c_1}.$

(3) If $\lambda^2-4\mu=0,$ then we obtain the rational solution

$$U(\xi)=\pm\frac{1}{2}\sqrt{\frac{k_2(k_1^2-\omega^2)(\lambda^2-4\mu)}{\omega}}\frac{c_2}{c_1+c_2\xi}, \quad (51)$$

where $\xi=k_1x+k_2y+\frac{\omega t^{\alpha}}{\Gamma(1+\alpha)},$

3.4. Example 4: The time fractional nonlinear Klein-Gordon-Zakharov equations

These equations can be written in the following nonlinear system [21]:

$$D_t^{2\alpha}u - u_{xx} + u + \beta_1 uv = 0,$$
$$D_t^{2\alpha}v - v_{xx} - \beta_2(|u|)_{xx} = 0,\ 0 < \alpha \le 1, \qquad (52)$$

With $u(x,t)$ is a complex function and $v(x,t)$ is a real function, where β_1, β_2 are nonzero real parameters. This system describes the interaction of the Langmuir wave and the ion acoustic in a high frequency plasma. Using the wave variable

$$u(x,t) = \phi(x,t)\exp\left[i\,(kx + \frac{\omega t^{\alpha}}{\Gamma(1+\alpha)})\right], \qquad (53)$$

where $\phi(x,t)$ is a real-valued function, k, ω are real constants to be determined and ξ_1 is an arbitrary constant. Then the system (52) is carried to the following PDE system:

$$D_t^{2\alpha}\phi - \phi_{xx} + (k^2 - \omega^2 + 1)\phi + \beta_1 v\,\phi = 0,$$
$$\omega D_t^{\alpha}\phi - k\,\phi_x = 0, \qquad (54)$$
$$D_t^{2\alpha}v - v_{xx} - \beta_2\phi_{xx} = 0.$$

Setting

$$v(x,t) = v(\xi), \quad \phi(x,t) = \phi(\xi),$$
$$\xi = \omega x + \frac{kt^{\alpha}}{\Gamma(1+\alpha)}, \qquad (55)$$

then we get

$$v(\xi) = \frac{\beta_2\omega^2\phi^2(\xi)}{(k^2-\omega^2)} + C, \qquad (56)$$

and

$$\phi'' + \frac{(k^2 - \omega^2 + \beta_1 C + 1)}{(k^2-\omega^2)}\phi + \frac{\omega^2\beta_2\beta_1}{(k^2-\omega^2)^2}\phi^3 = 0, \qquad (57)$$

where C is an integration constant, and $k \ne \pm\omega$. Balancing ϕ'' and ϕ^3, we get

$$\phi(\xi) = a_0 + a_1\left(\frac{G'}{G}\right), \qquad (58)$$

where a_0, a_1 are constants to be determined later, such that $a_1 \ne 0$. Substituting (58) into Eq.(57) and collecting all terms with the same powers of $(G'/G)^i, i = 0,1,2,3$ together, equating each coefficient to zero, we have the following system of algebraic equations:

$$0: \lambda\mu a_1 + \frac{a_0(k^2-\omega^2+\beta_1 C+1)}{(k^2-\omega^2)} + \frac{a_0^3\beta_1\beta_2\omega^2}{(k^2-\omega^2)^2} = 0,$$

$$1: \lambda^2 a_1 + 2\mu a_1 + \frac{a_1(k^2-\omega^2+\beta_1 C+1)}{(k^2-\omega^2)} + 3a_0^2\frac{a_1\beta_1\beta_2\omega^2}{(k^2-\omega^2)^2} = 0,$$

$$2: 3\lambda a_1 + 3a_0\frac{a_1^2\beta_1\beta_2\omega^2}{(k^2-\omega^2)^2} = 0,$$

$$3: 2a_1 + \frac{a_1^3\beta_1\beta_2\omega^2}{(k^2-\omega^2)^2} = 0.$$

Solving these algebraic equations yields

$$a_1 = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{-2}{\beta_1\beta_2}},\ a_0 = \frac{\lambda}{2}a_1,$$
$$\mu = \frac{(k^2-\omega^2)(\lambda^2-2) - 2(1+c\,\beta_1)}{4(k^2-\omega^2)},$$

(1) If $\lambda^2 - 4\mu > 0,$ then we have

$$\phi(\xi) = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{4\mu-\lambda^2}{2\beta_1\beta_2}} \times \left(\frac{c_1\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right) + c_2\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}{c_1\cosh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right) + c_2\sinh\left(\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right)}\right) \qquad (59)$$

Substituting Eqs.(8), (10), (12), (14) obtained by Peng [18] into Eq.(59), we have respectively the following Kink-type traveling wave solutions:

(i) If $|c_1| > |c_2|,$

$$\phi(\xi) = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{4\mu-\lambda^2}{2\beta_1\beta_2}} \times \tanh\left[\frac{\xi}{2}\sqrt{\lambda^2-4\mu} + \mathrm{sgn}(c_1c_2)\psi_1\right], \qquad (60)$$

(ii) If $|c_2| > |c_1| \ne 0,$

$$\phi(\xi) = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{4\mu-\lambda^2}{2\beta_1\beta_2}} \times \coth\left[\frac{\xi}{2}\sqrt{\lambda^2-4\mu} + \mathrm{sgn}(c_1c_2)\psi_2\right], \qquad (61)$$

Where $\psi_1 = \tanh^{-1}\left(\frac{|c_2|}{|c_1|}\right),\ \psi_2 = \coth^{-1}\left(\frac{|c_2|}{|c_1|}\right),$ and $\mathrm{sgn}(c_1c_2)$ is the sign function.

(iii) If $|c_2| > |c_1| = 0,$

$$\phi(\xi) = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{4\mu-\lambda^2}{2\beta_1\beta_2}}\coth\left[\frac{\xi}{2}\sqrt{\lambda^2-4\mu}\right], \quad (62)$$

(2) If $\lambda^2 - 4\mu < 0$ then we have the exact solution

$$\phi(\xi) = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{4\mu-\lambda^2}{2\beta_1\beta_2}}\times \left(\frac{-c_1\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}{c_1\cos\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)+c_2\sin\left(\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right)}\right) \quad (63)$$

$$\phi(\xi) = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{\lambda^2-4\mu}{2\beta_1\beta_2}}\tan\left[\xi_1-\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right], \quad (64)$$

$$\phi(\xi) = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{\lambda^2-4\mu}{2\beta_1\beta_2}}\cot\left[\xi_2+\frac{\xi}{2}\sqrt{4\mu-\lambda^2}\right], \quad (65)$$

where $\xi_1 = \tan^{-1}\frac{c_2}{c_1}, \ \xi_2 = \cot^{-1}\frac{c_2}{c_1}..$

(3) If $\lambda^2 - 4\mu = 0$ then we have the rational solution

$$\phi(\xi) = \frac{k^2-\omega^2}{\omega}\sqrt{\frac{-2}{\beta_1\beta_2}}\,\frac{c_2}{c_1+c_2\xi}. \quad (66)$$

Substituting $\phi(\xi)$ into (53) and (56) to get the solutions $u(\xi)$ and $v(\xi)$ of the original equations (52) .

4. Conclusions and physical meaning of the results

The (G'/G)-expansion method of the fractional partial differential equations is applied successfully for solving the time fractional nonlinear coupled Burgers equations, the time fractional nonlinear coupled KdV equations, the time fractional nonlinear Zoomeron equation and the time fractional nonlinear Klein-Gordon-Zakharov equations. As one can see, the nonlinear fractional complex transformation for ξ is very important, which ensures that a certain fractional nonlinear partial differential equation can be turned into another nonlinear ordinary differential equation of integer order whose solutions can be expressed by a polynomial in (G'/G) where G satisfies the linear ODE (8). We have given some figures to illustrate some of our results. The physical meaning of our results can be summarized as follows: The results (17), (44), (60) represent the kink-shaped soliton solutions. The results (18),(19),(45),(46),(61),(62) represent singular kink-shaped soliton solutions. The results (30) represents the bell -kink-shaped soliton solution , while the result (31) represents the singular bell -kink- shaped soliton solution.

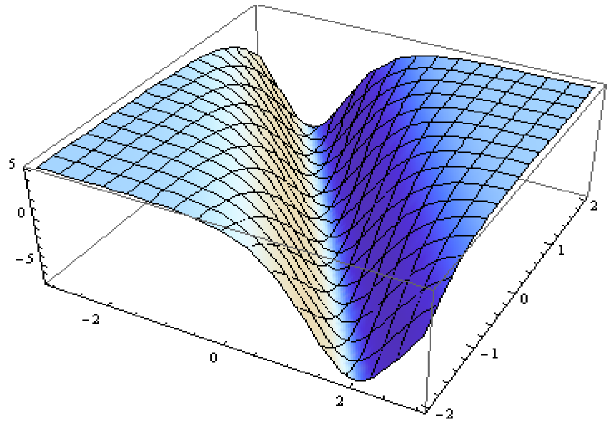

Figure 1: The plot of $U(\xi)$ in (30) when $a_0 = 1, \mu = 1, \lambda = \sqrt{8}, k = 1, \alpha = 1, \omega = 1, \mathrm{sgn}(c_1c_2)=0.$

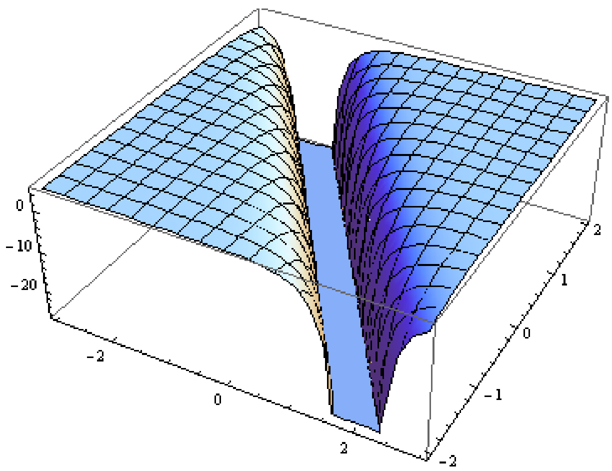

Figure 2: The plot of $U(\xi)$ in (32) when $a_0 = 1, \mu = 1, \lambda = \sqrt{8}, k = 1, \alpha = 1, \omega = 1.$

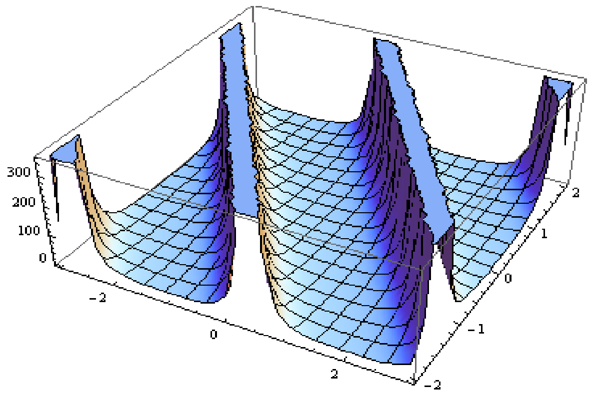

Figure 3: The plot of $U(\xi)$ in (35) when $a_0 = 1, \mu = 1, \lambda = \sqrt{8}, k = 1, \alpha = 1, \omega = 1.$

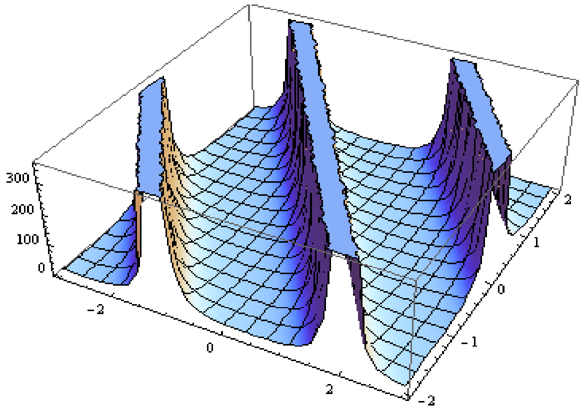

Figure 4: The plot of $U(\xi)$ in (36) when
$a_0 = 1, \mu = 1, \lambda = \sqrt{8}, k = 1, \alpha = 1, \omega = 1.$

References

[1] K. S. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, Wiley and sons, New York, 1993.

[2] A. A. Kilbas and H .M. Srivastava, J. J. Trujillo, Theory and applications of fractional differential equations, Elsevier, San Diego, 2006.

[3] B. Lu, “Bäcklund transformation of fractional Riccati equation and its applications to nonlinear fractional partial differential equations”, Phys. Lett. A, 376, 2045-2048, 2012.

[4] A. Golbabai and K. Sayevand, “The homotopy perturbation method for multi-order time fractional differential equations”, Nonlinear Sci. Lett. A, 1, 147-154, 2010.

[5] K. A. Gepreel, “The homotopy perturbation method to the nonlinear fractional Kolmogrov-Petrovskii- Piskunov equations”, Appl. Math. Lett., 24,1428-1434, 2011.

[6] S. Zhang, “A generalized exp-function method for fractional Riccati differential equations”, Commu. Frac. Calculus, 1, 48-51, 2010.

[7] B.Zheng , “(G'/G)-expansion method for solving fractional partial differential equations in the theory of mathematical physics”, Commun. Theor. Phys., 58, 623-630, 2012.

[8] K. A. Gepreel and S. Omran, “Exact solutions for nonlinear partial fractional differential equations”, Chin. Phys. B, 21, 110204-110210, 2012.

[9] G. V. Daftardar and S. Bhalekar, “Solving multi-term linear and nonlinear diffusion-wave equations of fractional order by Adomain decomposition method”, Appl. Math. Comput., 202, 113-120, 2008.

[10] G. V. Daftardar and H. Jafari, “Adomain decomposition: a tool for solving a system of fractional differential equations”, J. Math. Anal. Appl., 301, 508-518, 2005.

[11] G. Jumarie, “Cauchy’s integral formula via the modified Riemann-Liouville derivative for analytic functions of fractional order”, Appl. Math. Lett., 23, 1444-1450, 2010.

[12] G. Jumarie, “Fractional partial differential equations and modified Riemann-Liouville derivative new methods for solution”, J. Appl. Math. Computing, 24, 31-48, 2007.

[13] G. Jumarie, “Modified Riemann-Liouville derivative and Fractional Taylor series of non-differentiable func- tions: Further result”, Comput. Math. Appl., 51, 1367-1376, 2006.

[14] Z. B. Li and J. H. He, “Fractional complex transform for fractional differential equations”, Math. Comput. Appl., 15, 970-973, 2010.

[15] Z. B. Li, “An extended fractional complex transform”, Int. J. Nonl. Sci. Numer. Simula., 11, 335-337, 2010.

[16] A. J. M. Jawad , M. D. Petkovic´, A. Biswas, “Soliton solutions of Burgers equations and perturbed Burgers equation”, Appl. Math. Compu. ,216, 3370–3377, 2010.

[17] Y. Chen, H. L. An, “Numerical solutions of coupled Burgers equations with time- and space-fractional derivatives”, Appl. Math. Compu. ,200, 87–95, 2008.

[18] Z . Peng, “Comment on "Application of the (G'/G)-expansion method for nonlinear evolution equations"”, Commun. Theor. Phys., 52, 206-208, 2009.

[19] A. Atangana and A. Secer, “The time fractional coupled-Korteweg-de-Vries equations”, Abstract and Applied Analysis, Volume (2013), Article ID 947986, 8 pages.

[20] R. Abazari, “The solitary wave solutions of Zoomeron equation”, Appl. Math. Sci. 5, 2943 – 2949, 2011.

[21] Y. Shang, Y.Huang and W. Yuan, “New exact traveling wave solutions for Klein-Gordon equations”, Comput. Math. Appl. 56, 1441, 2008.

Prof. Dr. Elsayed M. E. Zayed
Professor of Mathematics (from 1989 till now)
at Zagazig University, Egypt. His interests are the nonlinear PDEs in Mathematical Physics, Inverse problems in differential equations, Nonlinear difference equations. He Published about 232 articles in famous International Journals around the world. He was the head of Mathematics Department at Zagazig University, Egypt from 2001 till 2006. He is an Editor of WSEAS Transaction on Mathematics. He has got some Mathematical prizes by the Egyptian Academy of scientific research and Technology.

He got the Medal of Science and Arts of the first class from the president of Egypt. He got the Medal of Excellent of the first class from the president of Egypt. He is one of the
Editorial Board of the Punjab University Journal of Mathematics. He is a member of the Editorial Board of the Int. J.PDEs and Applications. He also is a member of the Editorial Board of The SOP Transaction on applied Mathematics.
He have reviewed many articles for many international Journals . He has got his BSC of Mathematics from Tanta University, Egypt 1973 . He has got two MSC degrees in Mathematics .The first one from Al-Azher University, Egypt 1977, while the second one from Dundee University, Scotland, UK 1978. He has got his PHD in Mathematics from Dundee University, Scotland, UK 1981. He has got the Man of the Year award, (1993), by the American Biographical, Institute, U.S.A. He has got the Research Fellow Medal, (1993), by the American Biographical Institute, U.S.A. The Curriculum Vitae of Professor E.M.E.Zayed has been published by Men of achievement of the International Biographical center, Cambridge, vol. 16, (1995), p.534 The Curriculum Vitae of Professor E.M.E.Zayed has been published by Dictionary of International Biography of the International Biographical center, Cambridge, vol. 23, (1995), p.714.
He was an assistant Professor at United Arab Emirates University, Al-Ain (1987-1992). He was a Professor of Mathematics at Taif University, Saudi Arabia (2006- 2010).